The jaw is semilunar, narrow, thin, concentrically very finely, and radiately distantly and indistinctly, striated, the anterior concave edge is nearly perfectly entire.

The radula is long, moderately narrow, consisting of about 80 transverse, slightly angular series of teeth, there being 53 teeth in each series. The centre tooth is smaller than the adjoining, with a simple, inflected and pointed tip; it is contracted towards the base. The 14 inner laterals are longer and stronger than the 12 outer laterals. They are all tri-cusped; at first the median cusp is by far the largest, gradually, the lateral increase in size, while at the same time the median cusp decreases, until on the outermost lateral teeth the three cusps are almost equal. On the whole the form of the teeth agrees better with that of the Helicidæ than with the Zonitidæ. The dental formula is $12 + 14 — 1 — 14 + 12$.

On a quantitative method of testing a "Telegraph Earth,"—*by* W. E. Ayrton, Esq.

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The method that has been used up to the present time for testing a telegraph "earth" has been a qualitative method only, that is to say, although it may in a rough way have answered the question, is an "earth" good or bad, it was quite unable to give any answer to the question, how good or how bad.

In Europe the ordinary way to make an "earth" is to use the iron gas, or water pipes, but in most places in India such pipes do not exist, so that some large piece of metal has to be buried for this purpose. A coil of iron wire, a piece of an iron post, or a copper plate have been used at different times. Now as the nature of the ground in the immediate neighbourhood of this buried piece of metal greatly affects its electrical utility, it becomes a question of great practical importance to determine in absolute units the resistance of the "earth" used in each particular case.

The following method devised by Mr. Schwendler is at present in use in the Indian Telegraph Department.

Select two other earths which are neither in metallic connection with each other nor with the telegraph earth to be tested. Two iron telegraph posts near the office answer the purpose very well, only care must be taken that there is perfect metallic contact between the leading wire and the iron post in each case. In the dry season it would be advisable to pour water over the three "earths" used. Measure the resistance between each set of "earths," and in this way obtain three independent equations containing the three resistances of the three "earths," and the known resistances of the three leading wires going respectively from each "earth" to the testing arrangement. For instance calling x the resistance of the "earth" to be measured, that is, the resistance between the copper plate or iron wire (or whatever the "earth" consists of) and the ground, and α the known resistance of the wire leading from this "earth" to the testing arrangement, y and z the resistances of the other two earths, and β and γ the resistances of their leading wires we have—

$$x + y + \alpha + \beta = r_1$$
$$y + z + \beta + \gamma = r_2$$
$$z + x + \gamma + \alpha = r_3$$

From these three equations, eliminating y and z, we obtain

$$x = \frac{r_1 - r_2 + r_3}{2} - \alpha \ldots\ldots\ldots\ldots\ldots\ldots (\mathrm{I})$$

And the question would be completely solved, if earth circuits did actually behave as simple metallic circuits. This is, however, not the case. For in the first place an "earth" long used for telegraphic purposes frequently acquires a highly polarized state, giving rise to a current. Secondly if the "earths" used are not of the same material, for instance one an iron post and the other a copper plate, they will form a galvanic element with the ground giving rise to a current. Thirdly a real earth current may exist from terrestrial causes, and lastly the testing current itself polarizes the "earths." Consequently the measurement of the same set of earths taken successively with positive and negative currents will not agree, and they will differ from each other much, if the current, due to the "earths," is large in comparison with the testing current itself. It, therefore, becomes necessary to devise some method by which trustworthy tests may be made, and to see how from the tests the real resistances of the "earths" may be arrived at.

Before and after each set of tests note the whole, or a definite portion, of the current caused by the two earths under measurement, by simply joining the two earths together through a galvanometer and observing the deflection. If this deflection is practically the same before and after the two tests with reverse currents, the "earths" have not altered their electrical condition while being tested, and the two values obtained may be used for further calculation. In order to keep the electrical condition of the "earths" constant, by preventing them becoming polarised by the testing current, it is necessary to measure with only momentary currents.

The formula which gives the actual value of the resistance of a pair of earths from the two values obtained by testing with positive and negative currents depends, of course, on the kind of testing arrangement employed. For a Wheatstone's balance the formula is

$$r = \frac{BF(A+B)(W'+W'') + B^2\left\{A(W'+W'') + 2W'W''\right\}}{AB(W'+W'') + 2AF(A+B) + 2A^2B} \quad \text{(II)*}$$

where A and B represent the branch resistances in the bridge, A the resistance opposite to r the resistance to be measured, F the resistance of the testing battery, and W′ and W″ the resistances unplugged respectively in the comparison coil to obtain balance when testing with reverse currents. Putting A equal to B, or testing with equal branches we have

$$r = \frac{(2F+A)(W'+W'') + 2W'W''}{W'+W''+2(2F+A)} \;\ldots\ldots\ldots\; \text{(III)}$$

If W′ and W″ are very nearly equal, or small compared with A and F we have

$$r = \frac{W'+W''}{2} \; \dagger$$

If the instrument used be a differential galvanometer in which the two coils have equal resistance, but opposite magnetic momentum, then

$$r = \frac{(2F+G)(W'+W'') + 2W'W''}{W'+W''+2(2F+G)} \;\ldots\ldots\ldots\ldots\; \text{(IV)‡}$$

* (See Appendix I, p. 181). † (See Appendix II, p. 182).
‡ (See Appendix III, p. 183).

where G stands for the resistance of one of the coils of the galvanometer.

By formulæ (II) (III) or (IV) the resistances respectively between each set of earths can be correctly calculated, and these values being substituted for r_1 r_2 and r_3 in formula (I), we can find x the required resistance of the earth.

When a Wheatstone's bridge or differential galvanometer are not available the required resistance of the "earth" may be obtained in the following way by comparative deflections. For simplicity two leading wires only need be used, one just long enough to reach to the most distant "earth" of the three, and the other just long enough to reach the next distant.

Make the five following observations of deflections with the galvanometer, the same battery being used in all cases, and each test made with positive and negative currents and the mean taken.

I. When the galvanometer alone is in circuit: deflection $= a^\circ$.

II. When the two leading wires, and the galvanometer are in circuit: deflection $= b^\circ$.

III. When the Telegraph earth, one of the new earths, the two leading wires, and the galvanometer are in circuit: deflection $= c^\circ$.

IV. When the Telegraph earth, the other new earth, the two leading wires and the galvanometer are in circuit: deflection $= d^\circ$.

V. When the two new earths, the two leading wires, and the galvanometer are in circuit: deflection $= e^\circ$.

Then if the deflections are small, so that they are proportional to the currents, we have

$$x = \frac{G + F}{2}\left(\frac{a}{c} + \frac{a}{d} - \frac{a}{b} - \frac{a}{e}\right)^{*}$$

where x is the required resistance of the "earth" G and F ths known resistances of the galvanometer and battery respectively.

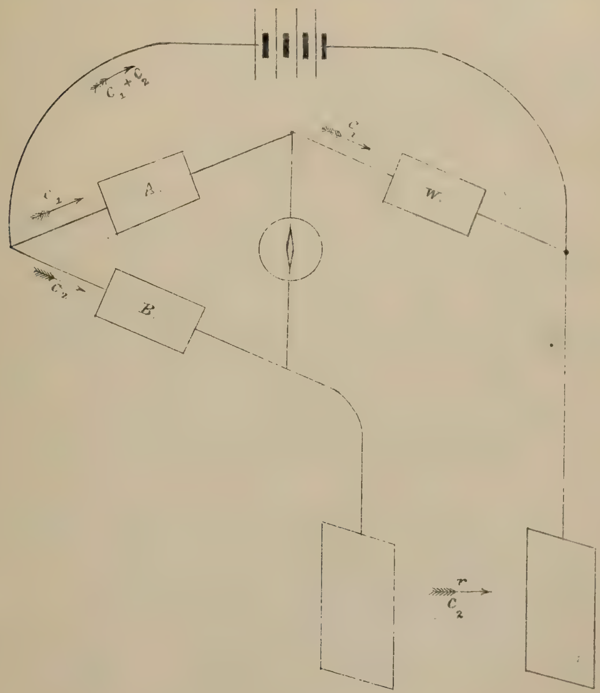
If the deflections be large and the galvanometer used by a sine or tangent galvanometer, then the sines or tangents respectively of the deflections must be substituted in the above formula instead of the simple deflections themselves.

* (See Appendix IV, p. 183).

(Appendix I.)

Equation (II) is necessarily precisely similar to that given by Mr. Schwendler in his "testing instructions" for finding the resistance of a line when a natural current exists in it; but as the proof, for brevity's sake, has been omitted there, I have given it as follows in its simplest form.

In the following figure, when balance is established, that is, when no current goes through the galvanometer, we have, by Kirchhoff's equations, when the earth current tends to help the testing current.



$$\left.\begin{array}{r} C_1 A - C_2 B = o \\ C_2 r - C_1 W' = e \\ (C_1 + C_2) F + C_2 (B + r) = E + e \end{array}\right\} \ldots\ldots\ldots\ldots\ldots (VIII)$$

where E is the electromotive force of the testing battery, and e that of the earth current.

If the testing battery be reversed so that the earth current tends to oppose the testing current we have—

$$\left.\begin{aligned} C'_1 A - C'_2 B &= o \\ C'_1 W'' - C'_2 r &= e \\ (C'_1 + C'_2) F + C'_2 (B + r) &= E - e \end{aligned}\right\} \ldots\ldots\ldots\ldots \text{(IX)}$$

From equations (VIII) by eliminating C_2 we obtain.

$$\frac{C_1 \frac{A}{B} r - C_1 W'}{\left(C_1 + C_1 \frac{A}{B}\right) F + C_1 \frac{A}{B} \left(B + r\right)} = \frac{e}{E + e}.$$

$$\text{or } \frac{\frac{A}{B} r - W'}{\left(1 + \frac{A}{B}\right) F + \frac{A}{B}\left(B + r\right)} = \frac{1}{\frac{E}{e} + 1} \ldots\ldots\ldots \text{(X)}$$

Similarly from equations (IX) we obtain—

$$\frac{W'' - \frac{A}{B} r}{\left(1 + \frac{A}{B}\right) F + \frac{A}{B}\left(B + r\right)} = \frac{1}{\frac{E}{e} - 1} \ldots\ldots\ldots\ldots \text{(XI)}$$

Eliminating $\frac{E}{e}$ from equations (X) and (XI) we obtain.

$$r = \frac{BF(A + B)(W' + W'') + B^2 \left\{ A (W' + W'') + 2 W' W'' \right\}}{A B (W' + W'') + 2 AF (A + B) + 2 A^2 B}$$

(Appendix II.)

First let W' and W'' be very nearly equal, that is,

$$\text{let } W'' = W' + dW'$$

$$\begin{aligned} \text{then } 2 W' W'' &= 2 W' (W' + dW') \\ &= W'^2 + (W' + dW')^2 - \overline{dW'}\rceil^2 \end{aligned}$$

$$\therefore\ 2 W' W'' = (W' + W'')^2 - 2 W' W'' - \overline{dW'}\rceil^2$$

$$\text{or } 2 W' W'' = \frac{(W' + W'')^2}{2}$$

in which the square of a differential only is neglected. Substituting this value for 2 W' W'' in equation (III) we obtain :

$$r = \frac{\frac{2\,(2\,F + A)\,(W' + W'')}{2} + \frac{(W' + W'')^2}{2}}{2\,(2\,F + A) + W' + W''}$$

$$\therefore\ r = \frac{W' + W''}{2}.$$

Secondly let W′ and W″ be both small compared with A and F, but W′ and W″ not necessarily equal to one another, then

$$r = \frac{(2\,F + A)\,(W' + W'')}{2\,(2\,F + A)} \text{ approximately,}$$

$$\therefore\ r = \frac{W' + W''}{2} \text{ approximately.}$$

(Appendix III.)

Equation (IV) can be obtained directly from equation (III) by substituting G for A, and this is precisely what would be anticipated since the law for a differential galvanometer, when the currents balance one another, must be precisely the same as that for a Wheatstone's Bridge at balance with equal branches; the two branches of the Bridge corresponding respectively with the coils of the differential galvanometer.

(Appendix IV.)

If x, y, z, be the resistances of the three "earths" used, and α and β the resistances of the two leading wires then.

$$a^\circ = \frac{M}{G + F} \quad \left\{ \text{where M is a constant depending on the battery power employed, and the delicacy of the galvanometer.} \right.$$

$$b^\circ = \frac{M}{G + F + \alpha + \beta}$$

$$c^\circ = \frac{M}{G + F + x + y + \alpha + \beta}$$

$$d^\circ = \frac{M}{G + F + x + z + \alpha + \beta}$$

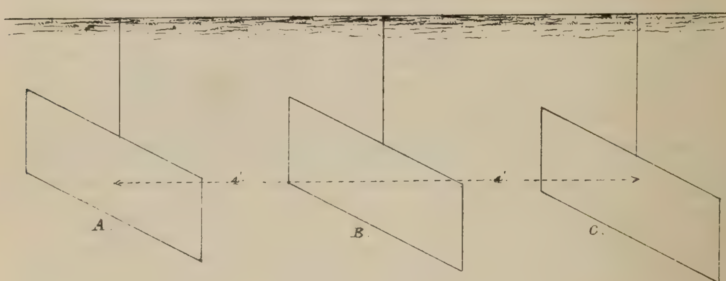
$$e^\circ = \frac{M}{G + F + y + z + \alpha + \beta}.$$

Eliminating y, z, $(\alpha + \beta)$, and M from the preceding five equations we obtain—

$$x = \frac{G + F}{2}\left(\frac{a}{c} + \frac{a}{d} - \frac{a}{b} - \frac{a}{e}\right).$$

As an illustration of the method described in this paper I made the following experiments with three earth plates, each 2 feet by 4, and using a Wheatstone's Bridge as my testing arrangement.

Experiment 1. The plates were buried vertically and parallel to one another with the longer edges horizontal and so that the line joining the centres of the plates was perpendicular to each plate. The centres were 4 feet apart, and 2 feet 6 inches below the surface of the ground.



The resistances of the circuit from plate A through the ground to plate C was very little more than that of the circuit from plate A through the ground to plate B although C was twice as far form A as B was, thus showing than in earth circuits the resistance is not so much in the earth itself, but exists between the surface of the metal plate and the earth. Using in the way previously explained in this paper the values I obtained I found that—

Resistance of plate A	=	8·49	Siemens	Units.
,, ,, B	=	5·86	,,	,,
,, ,, C	=	10·23	,,	,,

Experiment 2. The plates A and C remained as before, but B was placed horizontally, its centre being still 2 feet 6 inches below the surface of the ground, and 4 feet from the centres of A and B.

Resistance of plate	A	=	8·95 S. U
„ „	B	=	6·2 „ „
„ „	C	=	11·77 „ „

The values are somewhat higher than those previously obtained, but this I think is due to the ground being drier than before.

Experiment 3. Plates A and B remained as before, but B was placed horizontally, with its centre only 6 inches below the surface.

Resistance of plate	A	=	9·44 S. U
„ „	B	=	17·49 „ „
„ „	C	=	11·92 „ „

so that the resistance of B is nearly three times as great as it was before.

Experiment 4. All the plates in the same position as in the last experiment, but B was well surrounded with a layer of charcoal.

Resistance of plate	A	=	9·39 S. U
„ „	B	=	17·04 „ „
„ „	C	=	11·99 „ „

these results are very nearly the same as those obtained in experiment 3.

As a general rule, it may be said, that the lower an earth plate is buried the less is its resistance, or the better is it for telegraphic purposes. This rule, however, must be used with caution, because it assumes that the ground is of the same conducting quality at the different depths.

For example at the Jubbulpore office in order to obtain a good "earth" a large copper earth plate 5 feet square was buried vertically 17 feet deep. On my passing through Jubbulpore I tested this earth plate and found that it had a resistance of 50 S. U. On having the plate dug up I found that the high resistance was due to the plate having been buried in solid sandstone. I afterwards had the plate buried horizontally in the upper stratum of the soil and well surrounded with charcoal, and now the resistance was only 20 S. U, or less than one half of what it was before.

At Raneegunge Mr. Schwendler found that the earth plate there never had more than 0·5 S. U resistance. This very low resistance was probably due to the coal which exists there from 2 to 4 feet below the surface and in some places is actually exposed.